

Optimal Wind Farm Cabling

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Abstract—Wind farm cable length has a direct impact on the project cost, reliability and electrical losses. The optimum cable layout results in a lower unit cost of generating electricity offshore. This paper explores three cabling structures: the string structure, ring structures and multi-loop structure on a 3D seabed. The newly proposed multi-loop structure increases reliability and proves to be most economic when the failure rate and mean time to repair (MTTR) of cables are relatively high. Particle swarm optimization (PSO) is used to find the optimal substation location that minimizes the overall cable distance.

Index Terms—String structure, ring structure, multi-loop structure, 3D seabed, particle swarm optimization.

I. INTRODUCTION

THE UK is known to be one of the best locations for offshore wind power. The electricity that offshore wind farms generate accounts for 5% of annual demand of the UK, which is expected to increase to 10% by 2020 [1]. Despite the rapid development of offshore wind farms, recent wind farm projects have indicated that the cost has been stagnant at round £140 per MWh [2] and further reductions are very much dependent on infrastructure costs such as power cables and turbines and innovation in proactive maintenance. An effective way to reduce the cost is to find the optimal wind farm cable layout.

At present, the standard inter-array voltage is 33kV, and a common cabling design is a string structure, in which turbines are connected to a substation by strings of turbines. This structure has a relatively short cable length, but offers no redundancy. The reliability is enhanced in the ring structure. Ring structures often provide partial redundancy, e.g. When all turbines are generating the rated power output, a 67% rated ring can export at most 67% of the total rated power of the turbines connected to the ring in the case that a single cable outage occurs nearest to the substation. Partially rated rings are normally assumed for the 33kV baseline case because 100% rated rings are difficult to achieve at this voltage level due to the limited physical size of cable [3]. The proposed multi-loop structure can utilize the limited cable capacity to achieve full redundancy. This structure is more reliable and economical than the string and ring structure when the cable failure rate and MTTR are high.

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The cabling problem, which aims to minimize total cable length, can be approached as the Vehicle Routing Problem (VRP) in which turbines, substation and cable capacity are considered as customers, depot and vehicle capacity respectively. The buried cables are corresponding to the paths that vehicles travel. The ring structure cabling problem amounts to the classic Capacitated Vehicle Routing Problem (CVRP) which requires each vehicle to have a uniform capacity of a single commodity and all vehicles to start and finish at the depot. Various algorithms including exact, heuristic and metaheuristic can be used for the CVRP. According to a literature review by Baldacci et al. [4], the most effective exact algorithms dealing with this problem are Lysgaard et al.'s branch and cut [5], and Fukasawa et al.'s branch-and-cut-and-price [6]. However, such methods are impractical to address large wind farm problems, since they are computationally expensive. The heuristic Clarke and Wright's savings algorithm [7], is well-known for the CVRP, however, it does not include strategies for avoidance of cable crossings. Another heuristic method is called two-phase method [8]. It divides the problem into two steps. Phase one assigns customers to vehicles in several clusters respecting the limited vehicle capacity while Phase two connects customers of each cluster by using a traveling salesman problem heuristic. Metaheuristics such as the genetic algorithm by Barrier M [9], or the tabu search by Paolo Toth [10] can be applied to solve CVRP.

String structures correspond to open vehicle routing problems (OVRP), which do not require vehicles to return to the depot. Letchford [11] presented an exact algorithm for the OVRP, which is an extension of the branch and cut algorithm for CVRP. In this extension, the integer programming formulation and cutting planes are slightly modified to adapt a branch-and-cut code from CVRP to capacitated OVRP. This method can solve small to medium scale problems. Bauer and Lysgaard [12] presented a planar open saving heuristic algorithm which is an adaptation of the Clarke and Wright saving heuristic and can generate routing on average only 2% more expensive than the optimal routing. This saving heuristic, which is similar to the Clarke and Wright saving heuristic, calculates the distance saved by merging two string routes into one. The algorithm then selects the merges with the highest saving to generate a route respecting the cable capacity. The metaheuristic method in [13] is an improved genetic algorithm (GA) including a modified multiple traveling salesman problem to design radial arrays, which considers different cable cross sections and leads to a fast and effective result.

The problem of designing an optimal offshore wind farm collector system layout has been studied extensively in the literature. For instance, [14] presents a method of positioning

wind turbines and optimizing the wind farm direction for a regular shaped wind farm to balance energy yields and capital investment. [15] proposes three algorithms to design the optimal tree-structure cable layout of a collector system in a large-scale wind farm, aiming to minimize the total cable length. [16] presents an approach to determine the best connection structure of an offshore wind farm including string structure and ring structure, taking into account the cost of investment and the lost energy. Compared to previous research, the main novelties of this paper lie in the cabling on a rugged surface and the proposal of the multi-loop structures. Since a seabed is normally not a flat surface, this paper takes terrain into account, building a 3D seabed model to solve the cabling problem. It further presents a new cabling method for string structures and proposes the multi-loop structures for increased redundancy, considering the single cable type problem for wind farm cabling optimization with substation and turbine locations known. Additionally, it compares the economic efficiency of the optimal solutions for the ring structure, string structure and multi-loop structure under different cable failure rates and MTTRs. The optimal substation location is also investigated to minimize the overall cable length if only turbine locations are known. The algorithm used in this substation location problem is a particle swarm optimization algorithm (PSO), rather than the improved GA based solution approach in [17] and quasi-Newton Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm in [18]. The result shows the PSO algorithm can effectively find a suitable location for the substation within a small number of iterations.

II. CABLE

The inter-array cables are cables which connect the offshore turbines, via arrays to the common point of coupling. It is common practice to connect several (5-10) turbines together in an array, with each cable providing a link between two adjacent turbines. The cables used in a 33kV wind farm collector system are 3-core copper conductors with insulation/conductor screening and steel wire armoured. A number of cable sizes are available for such conductors [19]:

TABLE I
TYPICAL CABLE CHARACTERISTICS FOR 33kV CABLES

33kV Cable Type	Details		
	Overall Diameter(mm)	Weight(kg/m)	MVA(approx)
95mm ²	89	12.2	18
240mm ²	104	18.6	29
400mm ²	127	38	36
630mm ²	143	49	44

Cables providing a link between adjacent turbines are typically in the range of 500m to 950m while cables connecting wind turbine generator arrays to the substation can be longer, about 3km in length [19].

III. 3D SEABED AND GEODESIC ALGORITHM

The seabed has to be represented in 3D to reflect the true situation. Peaks function in Matlab is used to construct such a 3D seabed. For the seabed in Fig. 1, the central (green) point

and the 48 red points represent a substation and 5MW turbines respectively. The distance between neighboring turbines is about 8 units (or 800m). Finding the shortest distance between any two nodes requires the implementation of a geodesic algorithm since the surface is not flat. This paper uses an exact algorithm by Danil Kiranov [20]. The line in the figure is the shortest path between one turbine and the substation, which is found by this algorithm. Its length is 14.061 (1.4km).

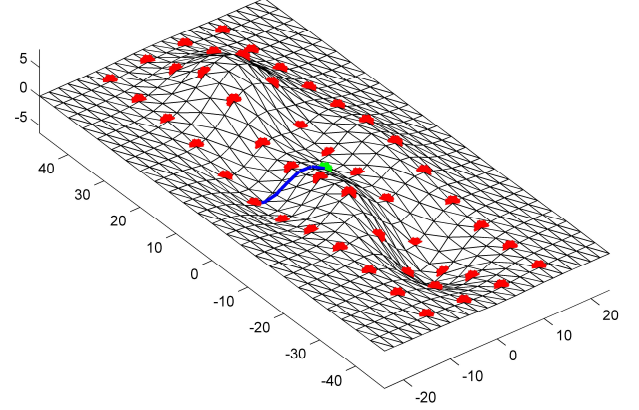


Fig. 1. 3D seabed.

IV. CABLING FOR RING STRUCTURES

A. Introduction of ring structures

When a cable failure occurs in a ring, the ring is divided into two operational strings which separately carry power. If the failure occurs in the cable nearest to the substation, the remaining cables are required to carry the total power of the whole ring. This is the worst case scenario and can overload the operating cables. For a 33kV base case, the largest inter-array cable size is taken to be 630mm², and 100% rated rings are very difficult to implement at this voltage due to the physical size of cable required [3] so a suitable partial rating is selected as a compromise between an achievable design and minimizing the lost energy.

B. Cabling methods for ring structures

The ring cabling problem can be considered a CVRP. A two-phase method is used to solve this cabling problem. Phase I applies the sweep algorithm which was presented by Gillett and Miller [21]. In this algorithm, the locations of customers and the depot are represented in polar coordinates, with a vector from the depot as a reference vector. This vector then sweeps either clockwise or counter-clockwise. As the vector sweeps customers, they are assigned to a cluster until the cluster reaches the vehicle capacity. The next customer will be assigned to a new cluster. The clustering is completed when all customers are assigned. As different reference vectors can result in different clusters, in this work 36 reference vectors, spaced apart by 10 degrees, are used to generate different clustering solutions. The total distances of all 36 solutions are compared and the layout with the shortest distance is chosen.

The sweep algorithm optimizes the use of the cable capacity, because clusters are formed until the full capacity is reached. It also naturally reduces the occurrence of cable crossings by clustering. One potential disadvantage is that the result may not

be optimal compared to a solution that does not use clustering.

After clustering, a route for each cluster is formed during Phase II by using Clarke and Wright savings algorithm [7] where the cost saving S_{ij} between two customers (turbines) i and j is calculated by merging two ring routes into one ring.

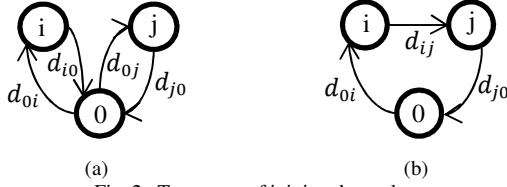


Fig. 2. Two ways of joining the nodes.

The depot (substation) is represented by node 0 and two customers (turbines) are represented by node i and node j . Fig. 2a describes that customer i and customer j are visited on separate routes, while the two routes are merged into one route going through both customer i and customer j in Fig. 2b.

Total distance in Fig. 2a:

$$D_a = d_{0i} + d_{i0} + d_{0j} + d_{j0}$$

Total distance in Fig. 2b:

$$D_b = d_{0i} + d_{ij} + d_{j0}$$

Cost saving between node i and node j :

$$S_{ij} = D_a - D_b = d_{i0} + d_{0j} - d_{ij}$$

The steps in Phase II of this algorithm are:

Step 1: Calculate cost savings S_{ij} for all pairs of customers in a cluster.

Step 2: Sort the cost savings S_{ij} in descending order.

Step 3: Generate an initial incomplete route by connecting the two nodes of the pair with the highest cost saving.

Step 4: If the next pair (i, j) of the descending cost savings list has one neighbor in the current route (assume the neighbor is i), then update the route by connecting node j to node i of the route. (For example, the current route is $a-c-...-k$. One element of the next pair (i, j) is the node a or k and the other element is not in the current route, then the pair has one neighbor to the route. Assume $i=a$, then the updated route is $j-a-c-...-k$).

Step 5: Go back to Step 4. Stop when all turbines in the cluster are connected.

Step 6: Proceed to the next cluster following Step 1 to Step 5, until all clusters have been processed.

C. Influence of different reference vectors

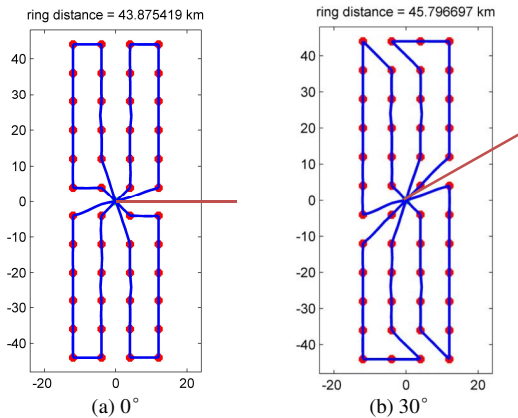


Fig. 3. Different layouts based on different vectors.

If the maximum turbine number in one ring (ring capacity) is set to 12, the turbines will be divided into four clusters. These clusters, however, vary with different reference vectors. Fig. 3a and Fig. 3b show the ring structures based on the reference vectors with 0° and 30° respectively. Only top views of these structures are shown for convenience. These two figures indicate that different reference vectors may cause a significant difference in the final cabling distance. For a fixed ring capacity, its optimal ring structure is the one with the shortest distance among the 36 structures with different vectors.

D. Results

In the 33kV base case, the largest inter-array cable size was taken to be 630mm² with capacity of 44MVA. As the wind turbine power factor is close to one at high wind speeds [22], the cable can carry current from up to 8 turbines. It should be noted that the charging currents of cables are not considered in this work as they only account for a small percentage (about 3%) of the rated currents [23]. To reserve capacity for redundancy, a ring can carry at most 15 turbines in this case. Hence the scenarios with ring capacity equal to and less than 15 turbines are analyzed. In this analysis 12 is the optimal ring capacity that leads to the shortest total distance, and its corresponding ring structure is 67% rated. This rating is also a suitable design option for the wind farm, which is verified in Section VII.

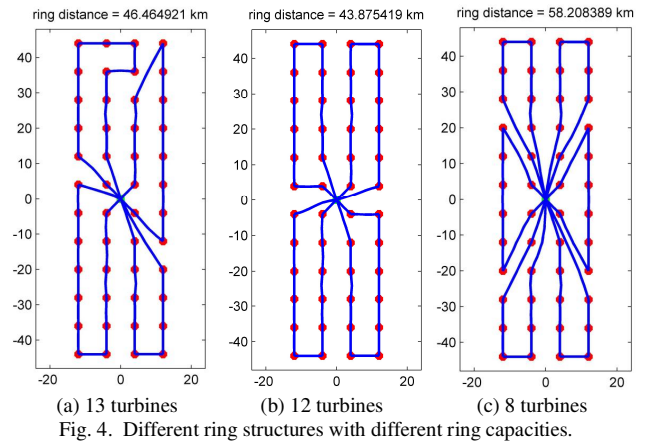


Fig. 4. Different ring structures with different ring capacities.

Fig. 4 shows that different ring capacities lead to various layouts with the layout of the optimal ring structure included. The optimal ring structure layout is also shown in Fig. 5 from a 3D view.

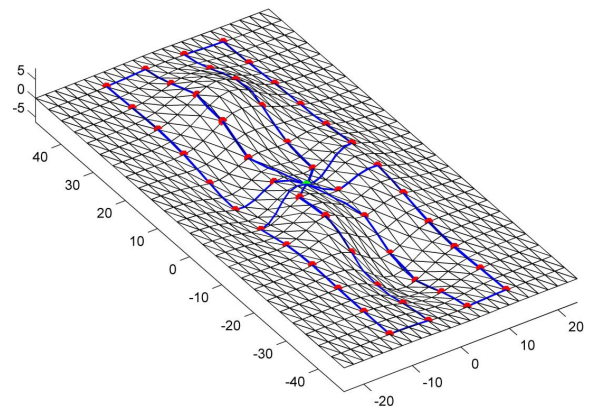


Fig. 5. Optimal ring structure.

V. CABLING FOR STRING STRUCTURES

A. Introduction of string structures

The maximum turbine number on a string is determined by the capacity of the cables. The worst case is a failure occurring in the cable nearest to the substation, which by nature carries the most turbines. Such a failure leads to a total loss of the turbines on the string.

B. Cabling Methods for string structures

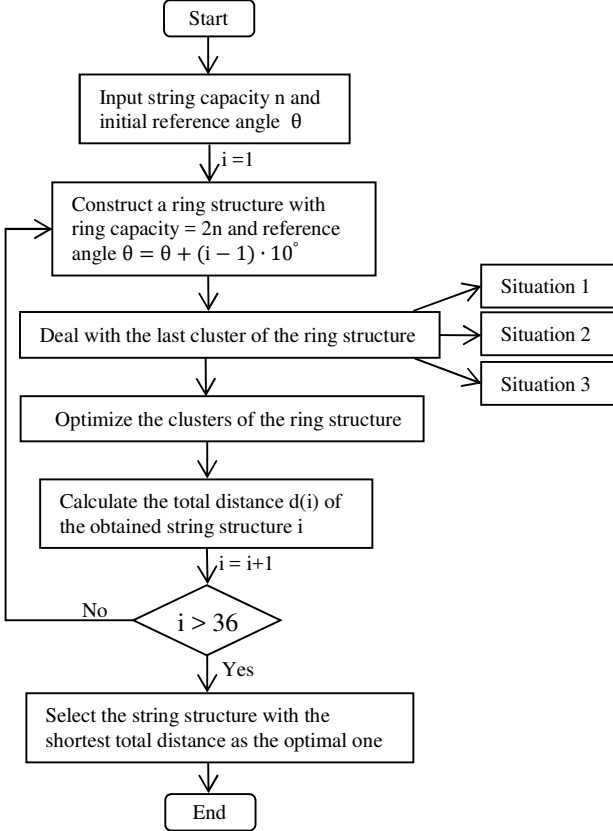


Fig. 6. Flowchart of cabling method for string structures.

The string structure cabling problem corresponds to an OVRP which does not require vehicles to return to the depot. This paper introduces a new heuristic constructive cabling method for string structures. Fig. 6 shows the flowchart of this method. Steps throughout the process are explained and illustrated in sub-sections 1) to 4):

1) Constructing a corresponding ring structure

In order to construct a string structure with string capacity n (string capacity is the maximum turbine number that a string can carry), firstly construct a ring structure with ring capacity equal to $2n$. Note that this ring structure could be based on any initial reference vector with angle θ . Next, to transform the ring structure into a string structure, the last cluster of the ring structure in particular requires consideration.

2) Dealing with the last cluster

A wind farm with m turbines and a ring capacity of $2n$, will have $m \bmod 2n$ turbines in the last ring, so the last ring can fall into three scenarios: 1) The number of turbines in the last ring is

exactly $2n$, which means the last ring can be dealt with in the same way as all other rings, 2) The number of turbines in the last ring is n or smaller, which means only one string, rather than two are required to export the power, 3) The number of turbines is smaller than $2n$ and larger than n , which means two strings will be required, but there is flexibility in how many turbines are allocated two each of the two strings. For all the three situations, the rings with $2n$ turbines are all cut into two strings with n turbines each.

Situation 1: To construct a string structure with string capacity 8, firstly build a ring structure with ring capacity 16 shown in Fig. 7a. Then all rings are divided into two strings carrying 8 turbines each, which is shown in Fig. 7b. This belongs to Situation 1. Note that the string capacity cannot exceed 8 as the maximum cable capacity is 8 turbines.

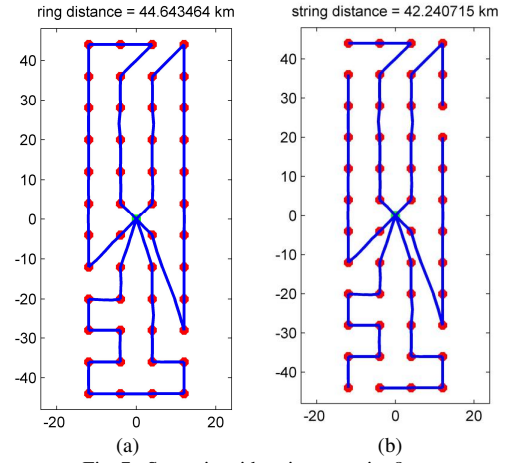


Fig. 7. Scenario with string capacity 8.

Situation 2: When the string capacity in this example is equal to 7 turbines, this case belongs to Situation 2, since the last ring includes 6 turbines. Fig. 8 shows the process of dealing with this situation. Each ring with 14 turbines is cut into two strings with 7 turbines each. As for the last ring, there are two methods of dealing with it.

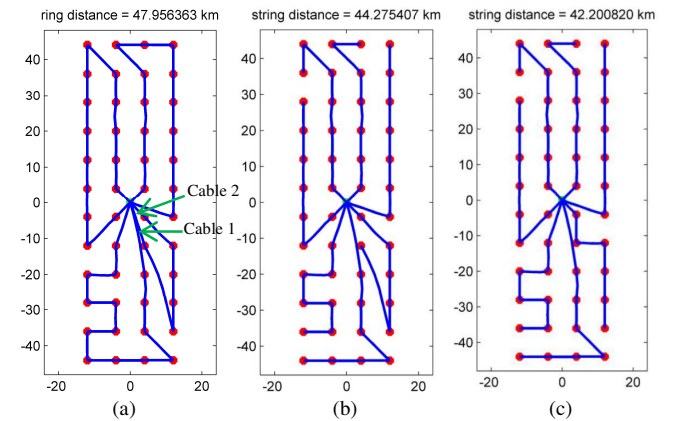


Fig. 8. Scenario with string capacity 7.

Method I: Cutting one cable. In order to transform the ring into a string, Cable 1 or Cable 2 as indicated in Fig. 8a can be cut. It is wiser to cut Cable 1 in this case since it is longer than Cable 2. The final string structure after cutting Cable 1 is shown in Fig. 8b.

Method II: Rerouting. The turbines in the last cluster could be connected as a string according to another saving algorithm in [12]. Clarke and Wright savings algorithm calculates the cost savings by merging two rings into one ring while this algorithm calculates the cost saving by merging two strings into one string. Fig. 9a shows that customer i and customer j are visited by two separate strings while Fig. 9b shows that these two routes are merged into one string route which goes through both customer i and customer j .

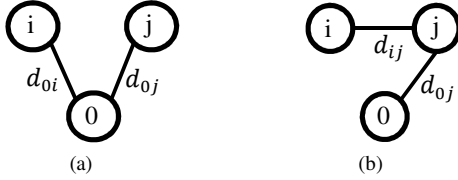


Fig. 9. Two ways of joining the nodes.

The cost saving is obtained by comparing their total distances:

$$S_{ij} = d_{0i} + d_{0j} - d_{ij} - d_{0j} = d_{0i} - d_{ij}$$

A large cost saving S_{ij} indicates that customer i should be visited immediately after customer j if vehicles leave the depot to customers. Note that S_{ij} is generally not equal to S_{ji} . The process of connecting customers is similar to that in Clarke and Wright's savings algorithm.

Fig. 8c shows that the six turbines in the last cluster are reconnected using Method II (the string saving algorithm). The string structure with the shorter distance is chosen from the results of both methods, in this case Method II. Note that the string saving algorithm does not necessarily result in a shorter distance than Method I. Fig. 10 is an example.

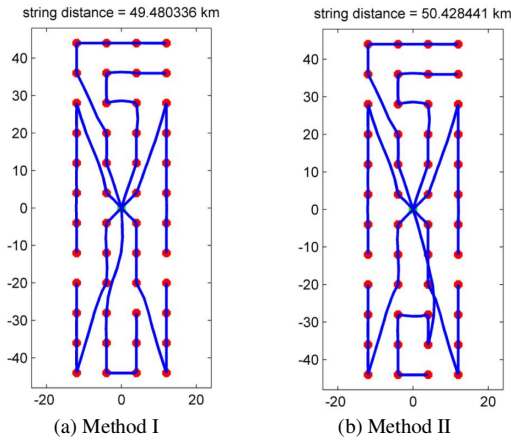


Fig. 10. Two methods for the last cluster.

Method II sometimes provides a strange string for the last cluster as shown in Fig. 10b. The string structure using Method I as shown in Fig. 10a has a shorter distance, so both methods have advantages in certain situations.

Situation 3: For a string capacity equal to 5, its corresponding ring structure consists of 5 rings with 10, 10, 10, 10 and 8 turbines respectively, which belongs to Situation 3. There are two methods to deal with the ring with 8 turbines, shown in Fig. 11.

Method I: Cutting one cable. Cable 1, Cable 2 and Cable 3 are admissible cables that can be cut without exceeding the string capacity. The admissible cables are obtained according to

the turbine number of the last ring and the string capacity. In this case, the longest cable among the three cables is cut so that the total distance of the two strings is shortest. The final string structure is shown in Fig. 11b.

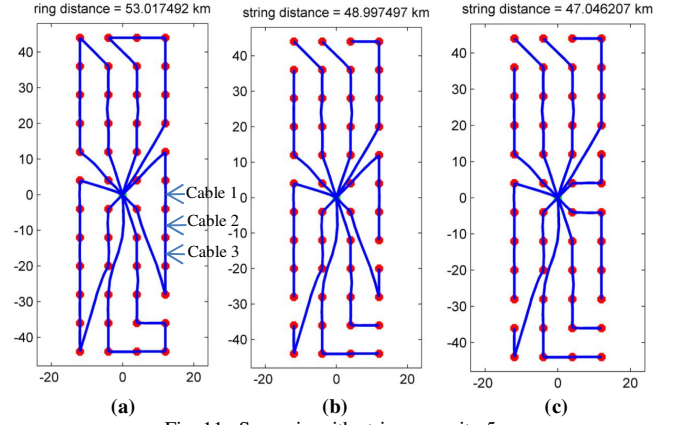


Fig. 11. Scenario with string capacity 5.

Method II: Cutting first and rerouting second. Cable 1, Cable 2 and Cable 3 are the admissible cables. Cutting any of these cables divides the ring into two strings, then the two strings are rerouted by the string saving algorithm. Cutting different admissible cables will lead to different results. In this case the string structure with Cable 1 cut as shown in Fig. 11c has the shortest distance among all three scenarios.

The solution for Situation 3 is the one with the shorter distance of the two string structures obtained by these two methods.

3) Optimizing clusters

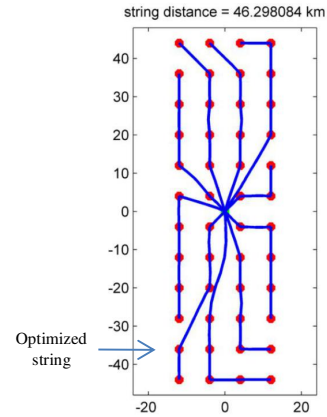


Fig. 12. The string structure after considering all clusters.

The methods in sub-section 2) only consider the optimization of the last cluster, however other clusters can be further improved as well. Take the structure in Fig. 11c for example, the eight strings obtained by cutting rings, each with 5 turbines, can be optimized using two possible methods. Method I is keeping the current route of each string, Method II is rerouting it using the string saving algorithm. The final solution for each string is the solution of the method leading to a shorter distance. Note that these two methods are also used to optimize the last cluster in Situation 1.

Fig. 12 shows the string structure with string capacity 5 after considering all clusters. Compared with Fig. 11c which only

optimizes the last cluster, Fig. 12 further optimizes the marked string. It is worth noting that the string structure in Fig. 12 is optimal only with respect to a certain reference vector and it may not be optimal among the 36 vectors.

4) Considering different vectors and string capacities

The layout of a string structure with string capacity n varies with different reference vectors. In this paper, 36 different vectors are considered and their corresponding total cabling distances $d(i)$ (i is from 1 to 36) are calculated. Among the structures with different vectors, the optimal string structure is the one with the shortest cabling distance.

C. Results

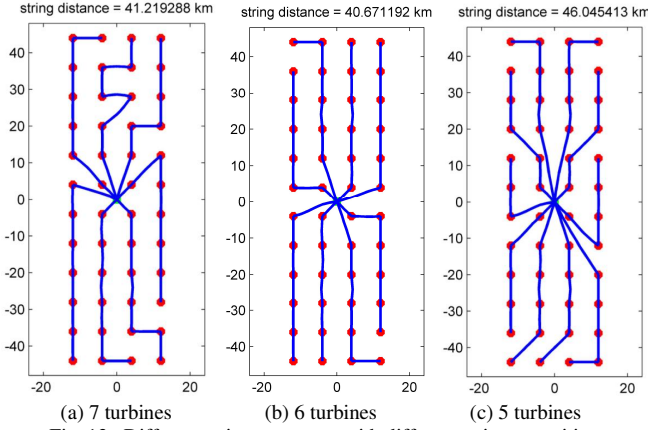


Fig. 13. Different string structures with different string capacities.

As the maximum cable capacity is 8 turbines, the structures with string capacity equal to or smaller than 8 should be analyzed. Fig. 13 shows the string structures with different string capacities. The results show that the optimal string capacity leading to the shortest distance is 6. In this case, the cables used could be 400mm² cables (36 MVA).

VI. CABLING FOR MULTI-LOOP STRUCTURES

A. Introduction of multi-loop structures

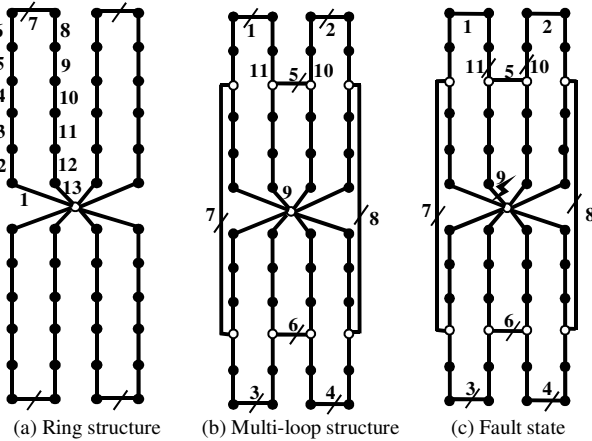


Fig. 14. Example of a multi-loop structure.

An example of a ring structure and a multi-loop structure is shown in Fig. 14, in which the diagonal lines indicate cable connections that are open. Fig. 14a and Fig. 14b show the normal states of the ring structure and the multi-loop structure

respectively. As for the ring structure, it is 67% rated because the cable could carry 8 turbines while there are 12 turbines in a ring. If Cable 13 fails, four wind turbines have to be out of service when the turbines reach their maximum capacity. As for the multi-loop structure, Cables 1 to 8 do not work in normal operating conditions. For a fault state such as the outage of Cable 9, Cable 10 and 11 will be opened and Cables 1, 2 and 5 will be closed to respond to the fault. In this case, there is no wind turbine out of service, indicating that this structure is fully redundant. It can be deduced that no generation is lost in the multi-loop structure during the period of a single cable failure.

B. Cabling methods for multi-loop structures

The method of constructing a multi-loop structure is based on a ring structure that is partially redundant. This process consists of two steps.

Step 1: Find connecting points. The white points in Fig. 14b represent the connecting points. A simple trick is applied to find such points. Assume that a ring includes n turbines and the cable capacity is m turbines ($m < n$). The connecting points are the $n+1$ -th turbines by counting the turbine number from the substation clockwise and anticlockwise. Fig. 14b shows that the white points in a ring are the ninth points according to the method.

Step 2: Connect the connecting points. Each ring in Fig. 14b has two connecting points. To connect these points, assume that a reference vector from the substation sweeps a circle either clockwise or counter-clockwise. During this sweep, a connecting point that is swept will be connected to the next swept connecting point if these two points are not in the same ring. Based on this method, Cables 5, 7, 6, 8 are routed to connect the points as shown in Fig. 14b.

C. Key points about multi-loop structures

- (1) If the turbine number of a ring is less than or equal to the cable capacity, the ring does not have connecting points and it is fully redundant. In this case, the ring can be neglected when constructing a multi-loop structure.
- (2) There is no relationship between an optimal ring structure and the optimal multi-loop structure. In other words, the optimal multi-loop structure is not directly obtained by constructing an optimal partially redundant ring structure. To construct a multi-loop structure for a ring structure with a certain ring capacity, the steps of finding the optimal multi-loop structure are: Firstly construct the ring structures with a variety of different reference vectors. Then construct the corresponding multi-loop structures based on these ring structures. Finally select the multi-loop structure with the shortest distance as the optimal one.
- (3) The third key point is about the condition of achieving full redundancy. Assume that there are x turbines, all operating at rated capacity in a ring, and the cable capacity is y . The worst case is that the cable nearest to the substation fails. In this case, $x-y$ (represented by z) turbines in the faulty ring have to be transferred to another operational ring. To carry z more turbines, the operational ring is divided into two strings, one with y turbines and the other with z turbines. The string with z turbines, which does not exceed the cable capacity, will carry the additional z turbines from the faulty ring. Hence the string of the good ring with z turbines will finally carry $2z$ turbines.

However, the turbine number of this string cannot exceed the cable capacity. That is:

$$2z \leq y$$

After simplifying the inequality, that is:

$$x \leq \frac{3}{2}y$$

Hence the multi-loop structure in Fig. 14 is a critical case of achieving full redundancy since $x = 12$ and $y = 8$, and 67% rated ring structures all belong to the critical case. In order to achieve full redundancy, the ring capacity is set to be a value not greater than 12 according to the inequality.

D. Results

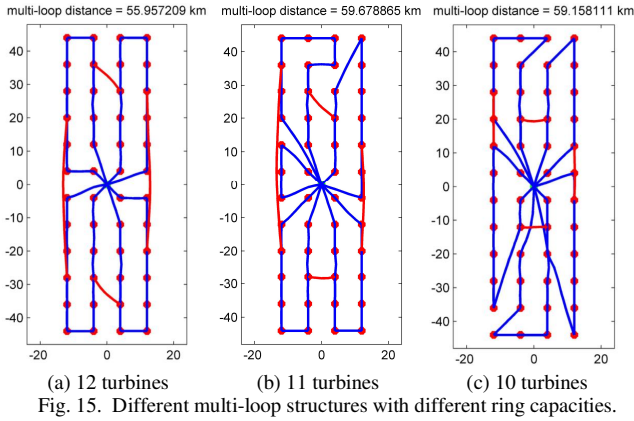


Fig. 15 shows different optimal multi-loop structures with different ring capacities. All three multi-loop structures can achieve full redundancy. The optimal multi-loop structure is the one with ring capacity 12 because it has the shortest total distance. It also has no cable crossing. In Fig. 15b, the ring with 4 turbines does not have connecting points, which is consistent with the first key point. In Fig. 15c, the layout has a cable crossing problem because there is a fully redundant ring that does not have connecting points. One cable has to pass through this ring to connect two partially redundant rings, which leads to the cable crossing problem.

VII. ECONOMIC ANALYSIS

Normally, to construct an optimal string structure for a wind farm, the total cabling distance is minimized without further considering redundancy. This implies that the layout in Fig. 13b is the optimal string structure. As for the optimal multi-loop structure, it is also the structure with the shortest distance, namely the one in Fig. 15a. Since multi-loop structures are fully redundant, the main difference in costs comes from cable length rather than lost generation cost. To determine the optimal ring structure, the total costs of the ring structures with different ring capacities are compared for a range of cable failure rate and cable MTTR listed in Table II [3]. The total cost comprises three distinct terms: cable expenditure, cable installation cost and lost generation cost. Other costs such as switchgear cost also varies with different structures. However, it is relatively small compared with the cable expenditure and installation cost [24] and is not considered in order to simplify the calculation. The comparison result is shown in Fig. 16.

TABLE II
PARAMETERS FOR TOTAL COST CALCULATION

Parameter	Value
Turbine Rated Power	5MW
Energy Price	£150/MWh
Cable Availability Failure Rate	Best = 0.0008 failures/km/annum
	Mid = 0.0094 failures/km/annum
	Worst = 0.015 failures/km/annum
Cable Availability MTTR	Best = 1 month
	Mid = 2 months
	Worst = 3 months
Wind Farm Life Time	25 years
400mm ² Cable Price	£250/m
630mm ² Cable Price	£350/m

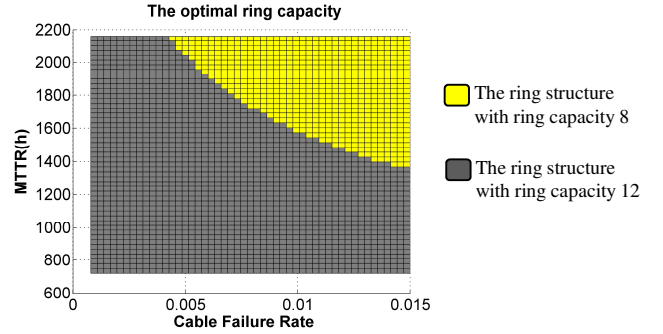
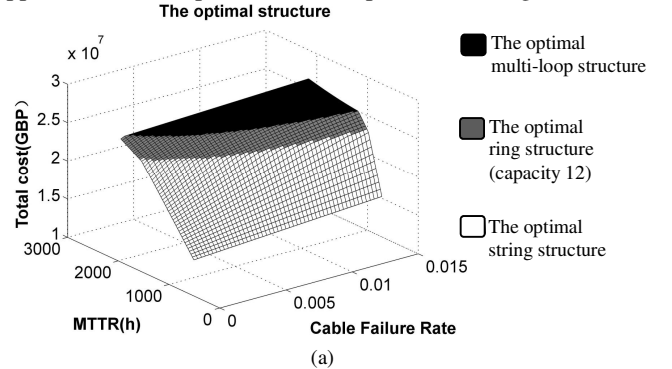


Fig. 16. The optimal ring capacity.

Fig. 16 shows that, depending on the MTTR and cable failure rate, ring capacity 8 or ring capacity 12 is optimal. The ring structure with ring capacity 8 is the one in Fig. 4c, which is fully redundant, while the ring structure with ring capacity 12 is the one in Fig. 4b, which has the shortest cabling distance and is 67% rated. The latter is the most economical structure when MTTR and cable failure rate are low, while the optimal choice is the former for high MTTR and cable failure rate. This figure also indicates that 67% rating is a suitable option for this wind farm since it accounts for a large area.

To determine the most economical structure for the wind farm, the total costs of four structures are compared. These structures are the optimal string structure, multi-loop structure and the ring structures with ring capacity 8 and 12. The calculation process of their total costs can be seen in the appendix. The comparison result is presented in Fig. 17.



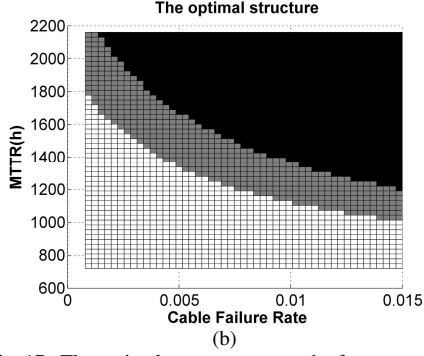


Fig. 17. The optimal structure among the four structures.

Fig. 17a shows the optimal structure among the four structures and its corresponding total cost under different cable failure rates and MTTR. Fig. 17b is the top view of Fig. 16a. In this figure, there is no yellow area representing the ring structure with ring capacity 8 since it is completely replaced by the multi-loop structure. This is due to the reason that they are both fully redundant but the multi-loop structure requires less cable, which can be verified by comparing Fig. 4c with Fig. 15a. The best choice is the string structure (white area) when both cable failure rate and MTTR are relatively low, while the best choice is the multi-loop structure (black area) when they are relatively high. The black area occupies a large proportion of the total area, indicating a potentially high application value of multi-loop structures.

The properties of multi-loop structures are summarized as follows: a multi-loop structure has the highest cable expenditure and cable installation cost among the three kinds of structures. It is more complex to control than a string or ring structure. Cable crossing can also be a concern in some multi-loop structures. However, multi-loop structures could solve the partial redundancy problem of ring structures due to the limited physical cable size, ultimately increasing the operational reliability of wind farms. The level of benefit of multi-loop structures is sensitive to the cable failure rate and MTTR. Additionally, a multi-loop structure may use less cable to achieve full redundancy than a ring structure. The idea of multi-loop structures can be utilized at all system voltages (33kV, 48kV and 66kV).

VIII. SITE SELECTION OF A SUBSTATION

The aim in this section is to determine the best substation location leading to the shortest cable length if only turbine locations are given. The algorithm used for finding the best location is the PSO algorithm.

As opposed to the classical optimization methods such as gradient descent and quasi-newton methods, particle swarm optimization does not require any information about the gradient of the objective function [25], which makes it especially suitable for solving this problem. This is because specific formulas of the total string distance, ring distance and multi-loop distance are non-trivial, while the values of the total distances for a given substation location can be easily obtained. The PSO algorithm is available in Matlab. As the 3D seabed model requires the time-consuming exact geodesic algorithm to calculate distances, for the substation location problem this

quickly becomes computationally expensive. To avoid this problem, a plane seabed is used for the study of the optimal substation location. This example assumes there are ten turbines in a wind farm. Different substation locations correspond to different total distances. Creating the objective functions for the string structure, ring structure or multi-loop structure is necessary to implement the PSO algorithm. In this PSO algorithm, the swarm size is set to 10, the substation location is represented by the coordinate $[a, b]$, leading to a particle size of 2 and a scope of $[x_{\min}, x_{\max}; y_{\min}, y_{\max}]$. The iteration number is set to 30.

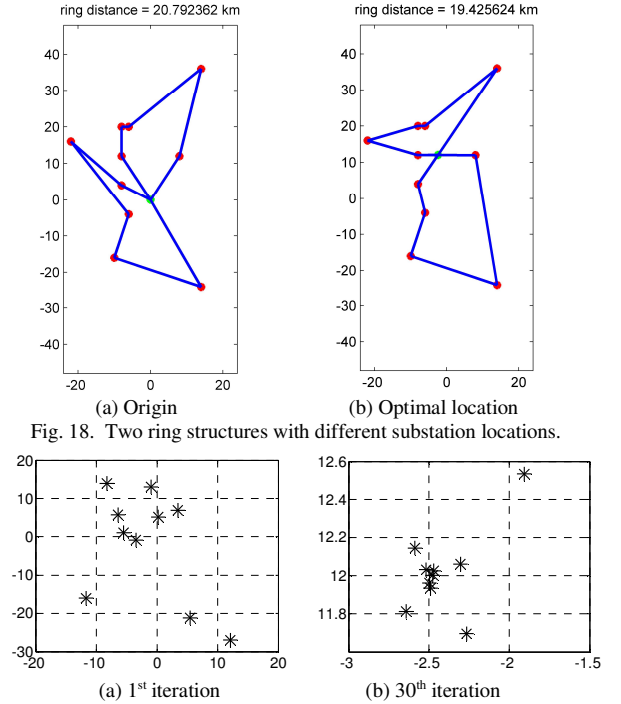


Fig. 18. Two ring structures with different substation locations.

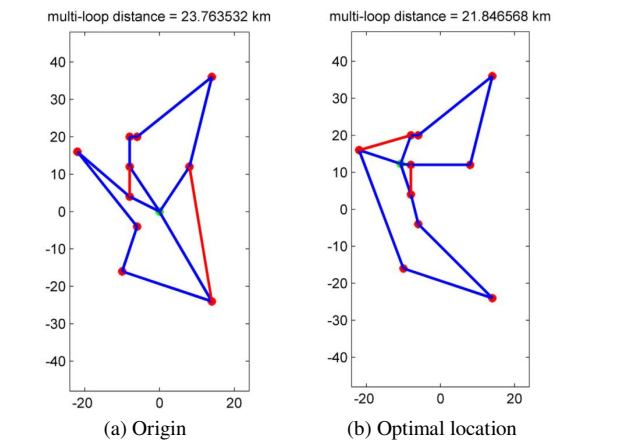


Fig. 20. Two multi-loop structures with different substation locations.

Assume the wind farm is built into a ring structure with ring capacity 5. Fig. 18a shows the optimal ring structure without optimizing the substation location, while Fig. 18b shows the optimal ring structure with the substation location calculated by PSO. The layout in Fig. 18b requires less cable than that in Fig. 18a, proving the effectiveness of the PSO algorithm. Fig. 19 shows the iteration process of the PSO with the particle scope

set to $[-20, 20; -30, 30]$. The 1st iteration is the process of initializing particles. After 30 iterations, the particles become much denser as the particle scope narrows down significantly. This is because the particles have the trend of moving towards the best position. The result shows that the optimal position is $[-2.46, 12.02]$ with a corresponding total distance of 19.43km. If the cable capacity is set to 4, the ring structure is not fully redundant. Fig. 20a and Fig. 20b show the corresponding multi-loop structures with the substation location $[0, 0]$ and the optimal substation location $[-10.86, 12.58]$, respectively.

When the iteration number increases from 30 to 50, the results remain almost the same, which implies that 30 is large enough to get a good solution. Hence the optimal substation location can be obtained within a small iteration number.

IX. CONCLUSION AND FUTURE WORK

The paper reports the following contributions:

- 1) The paper studies cabling on a 3D seabed that can simulate the realistic rugged seabed. Using a 3D seabed can obtain the specific cabling paths which can provide a better guidance on cabling. The disadvantage is that it is more time-consuming than studying a planar surface.
- 2) This work introduces a new heuristic constructive method for cabling string structures, which is based on the sweep algorithm, Clarke and Wright savings algorithm and string saving algorithm. The method naturally reduces the occurrence of cable crossings and can provide valuable solutions.
- 3) The paper presents a new cabling structure called multi-loop structure which can solve the partial redundancy problem due to the limited physical cable size in rings. Multi-loop structures provide enhanced reliability, and can save cost when cable failure rate and MTTR are above certain values. For a fully redundant wind farm, constructing a multi-loop structure may require less cable length than constructing a ring structure.

Future work will include avoidance strategies for cable crossings in multi-loop structures, benchmarking of results against alternative VRP algorithms and cabling in multiple cable scenarios.

APPENDIX

The cable installation cost varies with cable type and the nature of the seabed. The exact estimation of costs is project specific and has to be assessed on a case-by-case basis. Cable expenditure and cable installation cost normally account for about 15% and 4% of the total cost of a typical offshore wind farm, respectively [26]. Cable installation cost is estimated to be about 4/15 of cable expenditure. Note that a 400mm² cable is used for the string structure and a 630mm² cable is used for the ring structures and the multi-loop structure.

Cable expenditure = cable price · total cable length

Cable installation cost = 4/15 · cable expenditure

Lost generation cost of string structure:

$$L(a, k) = \text{MTTR} \cdot G \cdot k \cdot \eta \cdot p(a, k) \quad (1)$$

$$\text{TL} = \sum (L) \quad (2)$$

$$\text{Lost generation cost} = \text{TL} \cdot \text{pr} \cdot T \quad (3)$$

where

$L(a, k)$ represents the lost generation of the a -th string when k turbines are lost on the string;

MTTR is mean time to repair (h);

G is turbine capacity (MW);

η is the capacity factor;

$p(a, k)$ is the probability of losing k turbines on a -th string;

TL is the lost generation of the whole string structure per year, which is the sum of all the elements in L ;

pr is the energy price (£150/MWh);

T is wind farm life time (25 years).

The capacity factor η is calculated as follows:

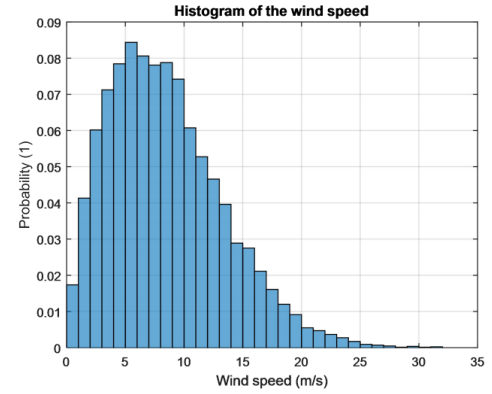


Fig. 21. Histogram of the wind speed.

$$P_w = \frac{1}{2} \rho v_1^3 A C_{pmax} \quad (4)$$

ρ the density of air (1.225kg/m³), v_1 the wind speed, A the surface swept by the rotor blades, C_{pmax} the maximum power coefficient and P_w the output power.

The rotor diameter of a 5MW turbine is normally 135m. C_{pmax} is set to 0.49 and P_w is 5MW then the rated speed v is calculated to be 10.5m/s.

Fig. 21 shows the probability distribution of wind speeds. Assume the cut in speed is 3.5m/s and the cut out speed is 25m/s. According to (4) and the wind speed distribution, the capacity factor η is calculated to be about 49.8%.

Lost generation cost of ring structure:

Situation 1: Only one cable fails in a ring.

A ring is cut into two strings for a single cable failure. If the turbine number on a string exceeds the cable capacity, calculate the mean curtailed energy when the output power of the turbines on the string exceeds the cable capacity. The wind speed distribution in Fig. 21 determines the mean of curtailed power. The total lost generation cost under Situation 1 is represented by $S1$.

Situation 2: Two cables fail in a ring.

If two cables fail, the ring turns into three strings, with two strings connected to the substation and one string disconnected. The lost generation of the disconnected string is calculated according to (1) where $p(a, k)$ is the probability of losing two cables and k is the number of turbines on the disconnected string. The calculation of the lost generation of the two connected strings is the same as in Situation 1. The total cost of lost generation in Situation 2 is represented by $S2$.

Situation 3: Three or more cables fail in a ring.

This situation can be neglected since the probability is very small. The total lost generation cost is $S1 + S2$.

Lost generation cost of multi-loop structure:

No generation is lost in a multi-loop structure during a single cable failure in a ring. Compared with the ring structure, the generation loss is also decreased significantly during a double cable failure, due to the increased redundancy. The generation loss of the multi-loop structure is estimated to be about one third of the loss of the ring structure under Situation 2.

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